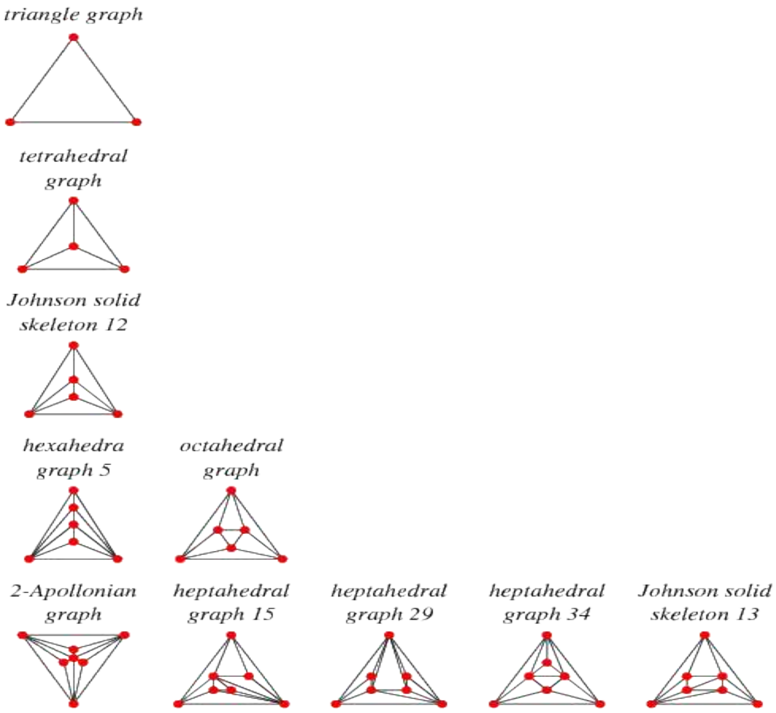
Four-coloring Problem

“No more than four colors are required to color the regions[vertices] of the map[graph] so that no two adjacent regions[vertices] have the same color.”[1]

Definition[2]

A planar graph G is said to be triangulated(also called maximal planar) if the addition of any edge to G results in a nonplanar graph.



Let Σ_n be a set of all maximal planar graphs which have n vertices and let $\Sigma_n = \left\{S_{n,1}, S_{n,1}, \cdots, S_{n,x_n}\right\}$.

The following statement is trivially true.

> “For arbitrary planar graph G which has m vertices, $\exists S_{m,i}$ $s.t.$ G is a subgraph of $S_{m,i}$”

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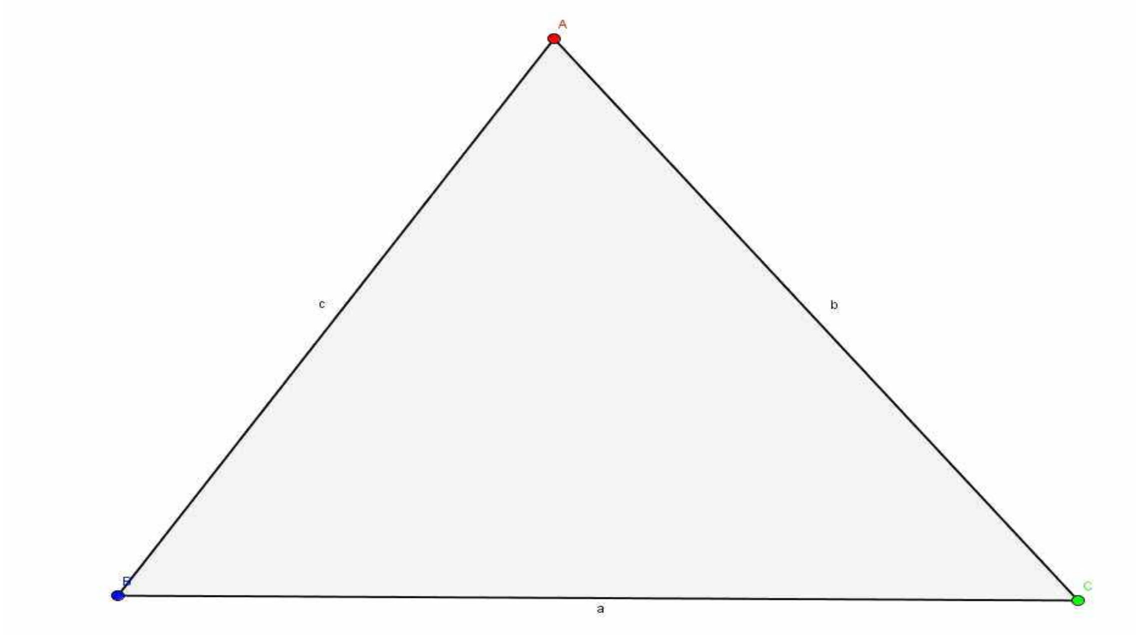
[1] https://en.wikipedia.org/wiki/Four_color_theorem / 2016. 6. 17.

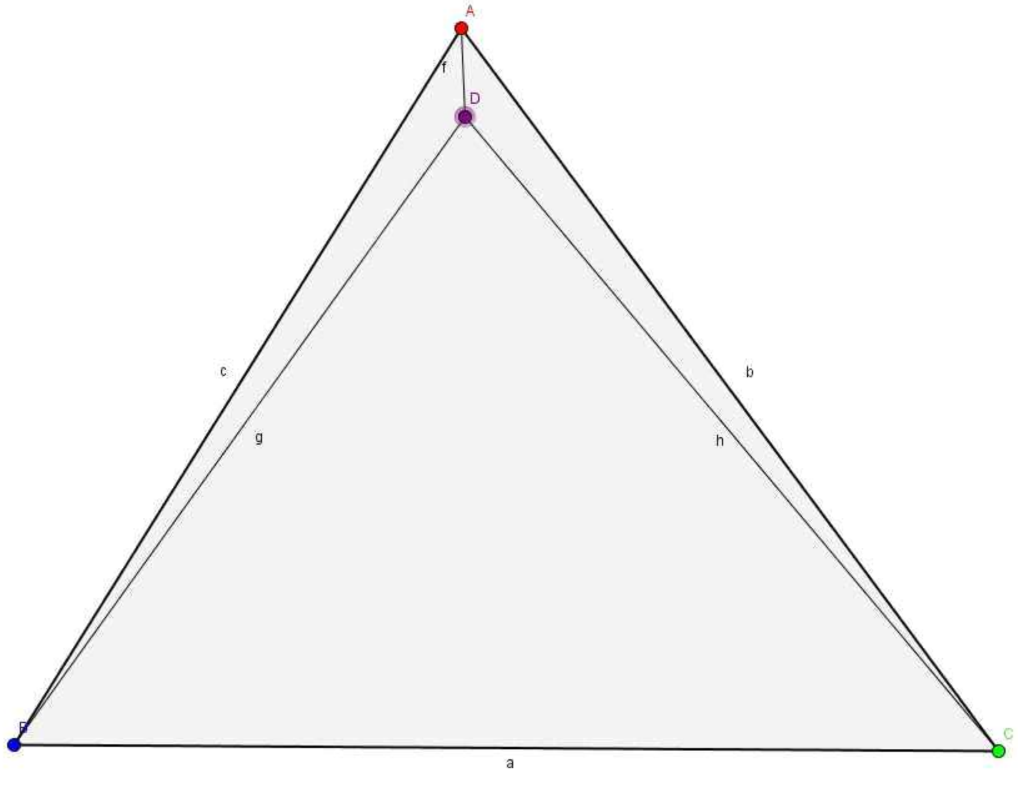
[2] http://mathworld.wolfram.com/TriangulatedGraph.html / 2016. 6. 17.
Consider Kuratowski's Theorem.

Because G is a subgraph of $S_{m,i}$, if two vertices is adjacent in G, then they are also adjacent in $S_{m,i}$. So it is sufficient to show that every maximal planar graph is four−colourable.

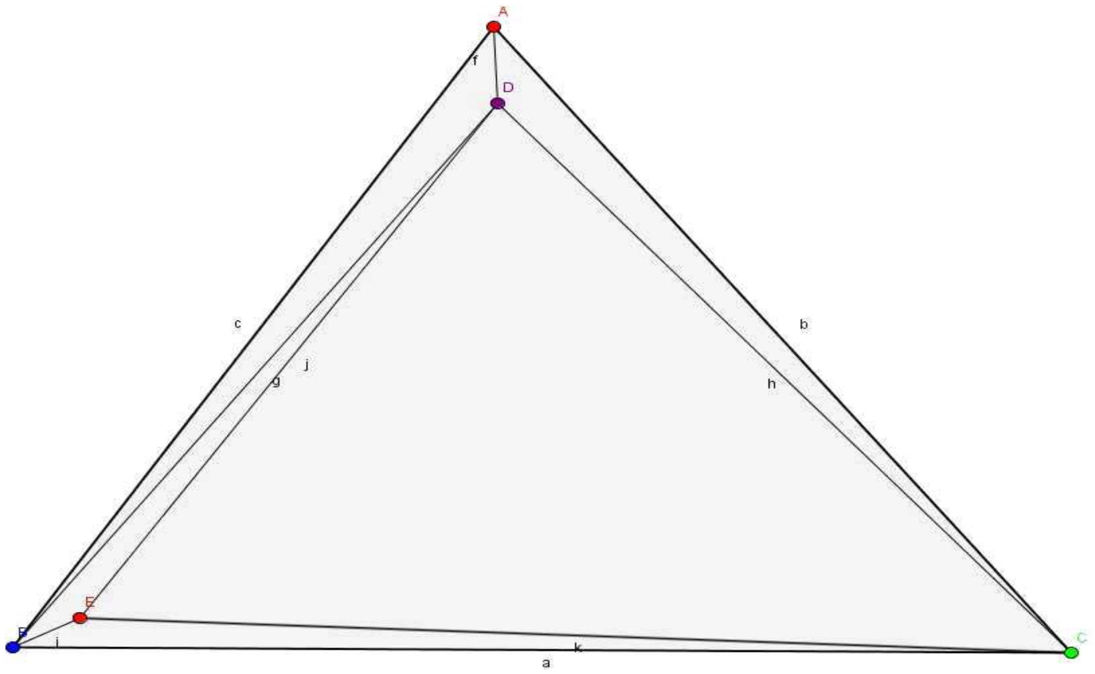
Way to coloring maximal planar graphs



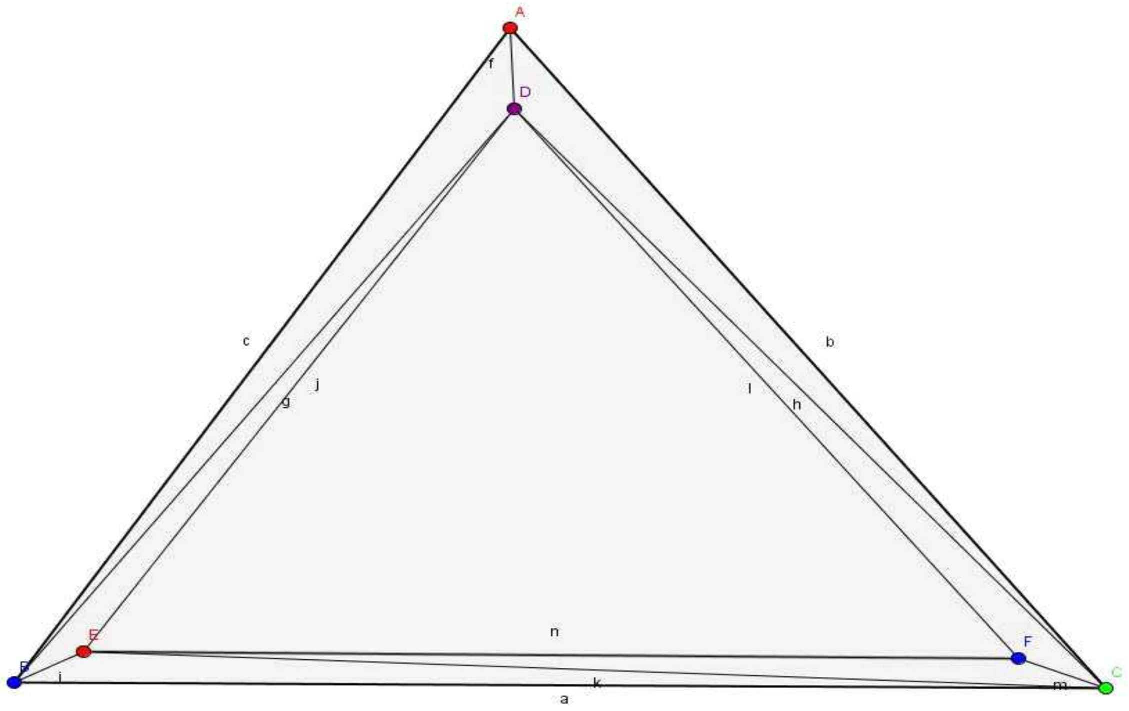
First, take three color to give initial three points.



Second, add new point and give new color.



Third, add new point and give theF color which is not surrounding that point.



In this way we can easily coloring every maximal planar graph with just four colors.

It means that every planar graph is colored by at most four colors.